

A stochastic mixing model for fibre distribution in fibre-reinforced concrete and its implications

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Abstract

In fibre-reinforced concrete, the distribution of fibres is generally assumed to be uniform, while their orientation is considered random. The spatial distribution of fibres can be effectively described using stochastic models. Under normal-consistency concrete conditions, fibre orientation may be assumed to be random; however, in the vicinity of the formwork, a certain degree of preferential alignment can be observed.

The most widely used method for determining material parameters is the three-point bending beam test. Owing to the small reference cross-sectional area, this test exhibits considerable scatter in the measured results. This variability is predominantly governed by the number and spatial position of fibres intersecting the fracture section. From a probabilistic viewpoint, this scatter is largely a statistical consequence of the limited fracture surface and the stochastic nature of fibre dispersion, rather than solely a material inconsistency. During evaluation, incorporating the fibre moment contribution leads to material parameters that are more representative of actual structural behaviour.

In real structural elements, the flexural load-bearing capacity of a cross-section is mainly governed by the same two factors, namely the number and position of fibres within the section. As the cross-sectional area increases, the relative statistical variability decreases, and the behaviour approaches the deterministic limit defined by the mean fibre distribution. Moreover, on-site fibre dosage control can be performed by evaluating measured fibre counts against a confidence interval derived from a stochastic mixing model at a prescribed confidence level.

To address these aspects, this paper introduces a novel stochastic mixing model for fibre distribution and demonstrates its applicability to material parameter evaluation, structural capacity assessment, and on-site quality control.

Keywords

Mixing model, stochastic modelling, fibre distribution, fibre-moment, quality control

1 Introduction

One of the main limitations in the application of fibre-reinforced concretes is the large scatter observed in the experimentally determined material parameters (Bernard, 2002; Parmentier, 2008; di Prisco et al, 2009; Sanal, I., Zihnioglu, 2013). This pronounced variability arises partly from the small reference volume of the test specimens (Juhász, 2025), and partly from the intrinsic scatter caused by the random spatial distribution of fibres (Cavalaro and Aguado, 2015). To obtain results with sufficient accuracy, a significantly larger number of specimens than those typically used in conventional concrete testing is required; accordingly, the relevant EN 14651 (2005) standard prescribes a minimum of 12 test specimens. Owing to the large scatter, the statistically derived characteristic values of the material parameters are often very low and, in some cases, may even become negative, rendering them unusable from an engineering perspective (Juhász, 2020).

Several approaches exist to mitigate the effect of large scatter. One option is to modify the specimen size in order to increase the reference volume or fracture surface. A representative example is the ASTM RDP test, in which the fracture surface area is significantly larger than that of standard beam tests. Consequently, the scatter of the test results is considerably reduced; moreover, further increasing the specimen size leads to an additional and substantial reduction in scatter (Bernard, 2013). Another approach involves the statistical evaluation of test results (Cavalaro and Aguado, 2015), as well as modified evaluation methods that explicitly account for the actual fibre positions within the specimen (Juhász, 2020).

It should be noted, however, that the pronounced scatter and the resulting low characteristic values are primarily associated with small-scale specimens. For larger reference volumes or cross-sectional areas, the scatter is significantly reduced. To account for this effect, certain guidelines recommend the use of mean test values (TR34, 2016) or allow for reduced partial safety factors when considering larger structural elements (*fib* Model Code, 2010; Richtlinie Faserbeton, 2008). The present paper aims to contribute recommendations for the further development of such approaches.

2 Intrinsic scatter due to fibre dispersion

In FRC, the distribution of fibres is characterized by the position of the centroid of fibres (O_f). The orientation of the fibres within a specified coordinate system is defined by their polar coordinates: α and γ (Fig. 1a). The proposed mixing model assumes a uniform dispersion of fibres, while the orientation of fibres remains uniform and stochastic. Local boundary effects (e.g. wall-effects) are neglected for standard beam dimensions, as their influence on global fibre statistics has been shown to be minor (Juhász, 2025). This model enables the determination of the number of intersection points of fibres across the cross-sectional area.

2.1 Distribution of fibres in the matrix

The number of fibres in a given volume V_0 is N_0 (Fig. 1b). Let us introduce $N_d [1/m^3]$ as the dosage value, then $N_0 = N_d V_0$. The probability of a single fibre being in a volume V ($V \in V_0$) is:

$$P_1 = \frac{V}{V_0} \quad (1)$$

The probability that n fibres are located within a volume V can be described using a hypergeometric distribution. However, for large populations, where the sample size is small compared to the total population, the hypergeometric distribution can be approximated by a binomial distribution as follows:

$$P_{1,n} = \binom{N_0}{n} P_1^n (1 - P_1)^{N_0 - n} \quad (2)$$

with mean value (m_1), standard deviation (σ_1) and coefficient of variation (CV) given by:

$$m_1 = N_1 = P_1 N_0 = N_d V \quad (3)$$

$$\sigma_1 = \sqrt{N_0 P_1 (1 - P_1)} \quad (4)$$

$$CV = \frac{\sigma_1}{N_1} = \frac{1}{\sqrt{N_0}} \sqrt{\frac{V_0}{V} - 1} \quad (5)$$

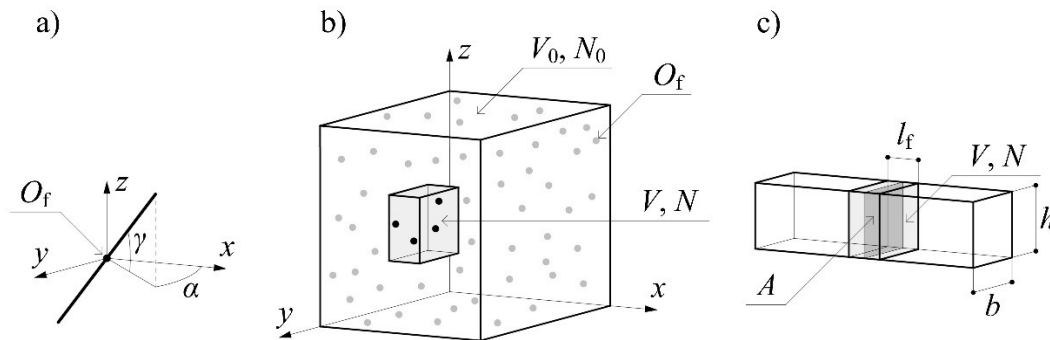


Figure 1: Distribution of fibres in the initial and reference volume (a) fibre local coordinate system, (b) V_0 and V , (c) interpretation of V in a beam specimen.

Let the fracture cross-section be A , which is assigned to volume V as shown in Fig. 1c, where l_f denotes the fibre length. The fibres within volume V can intersect cross-section A . Consider a standard-sized beam with $b = h = 150$ mm and $l_f = 50$ mm. In Fig. 2a, the relative variability is shown as a function of the V/V_0 ratio for different dosages. When $V_0 = V$, the relative variability becomes zero, as the number of fibres in V is equal to the total number of fibres in V_0 . The volume of a standard beam is $V_{\text{beam}} = 150 \times 150 \times 600$ mm, and the corresponding relative variability is plotted in the diagram. It is clear that even for a beam volume, the relative variability approaches the maximum value. In practical terms, during the production of concrete beams, the maximum relative variation can be considered, which depends solely on the amount of fibres added and not on the total volume of mixed concrete. The values of maximum coefficient of variation (CV_{max}) as a function of the dosage in case of V_{beam} are plotted in Fig. 2b. If V_0 tends towards infinity, CV_{max} and the corresponding standard deviation can be determined as follows:

$$CV_{\text{max}} = \lim_{V_0 \rightarrow \infty} \frac{1}{\sqrt{N_d V_0}} \sqrt{\frac{V_0}{V} - 1} = \frac{1}{\sqrt{N_d V}} \quad (6)$$

$$\sigma_{1,\text{max}} = CV_{\text{max}} N_1 = \sqrt{N_d V} \quad (7)$$

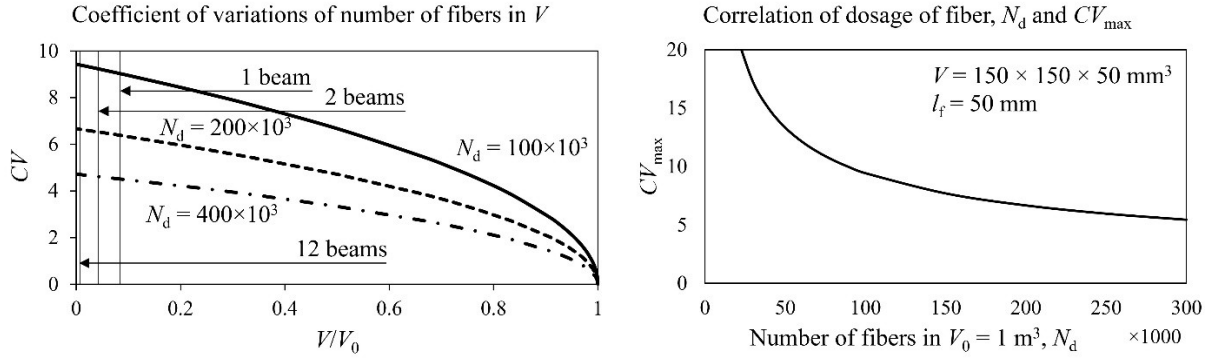


Figure 2: Coefficients of variation
(a) CV as a function of V/V_0 (b) CV_{max} as a function of fibre dosage

The coverage interval (90% probability interval) of fibre quantity can thus be determined as follows:

$$N \in [N_1 - 1.645 \sigma_{1,max}; N_1 + 1.645 \sigma_{1,max}] \quad (8)$$

Furthermore, when the expected number of fibres is sufficiently large, the binomial distribution may be approximated by a normal distribution in accordance with the central limit theorem. Accordingly, the random variable N is assumed to follow a normal distribution with parameters m_1 and σ_1 . In practical applications, σ_1 may be conservatively replaced by its maximum value, $\sigma_{1,max}$.

2.2 Cross-sectional intersection of fibres

The probability of fibre intersection within volume V is 0.5, as reported in several studies in the literature (Naaman, 1972; Krenchel, 1975; Stroeven, 1978). The influence of the wall effect is already negligible for specimens of standard beam dimensions (Juhász, 2020b) and is therefore disregarded in the present analysis. The probability of fibre intersection follows a binomial distribution; therefore, the expected number of fibres intersecting the cross-section is:

$$m_2 = N_2 = P_2 N_1 = 0.5 N_1 = 0.5 N_d V \quad (9)$$

and the standard deviation of this is:

$$\sigma_2 = \sqrt{N_2 P_2 (1 - P_2)} = \sqrt{0.25 N_2} = \sqrt{0.125 N_d V} \quad (10)$$

2.3 Model formulation

The number of fibres intersecting the cross-section, given the fibre length and dosage, can also be determined in a single step by combining the distribution parameters into a single expression. In this case, the expected number of fibres intersecting the cross-section is:

$$N_3 = m_3 = P_1 P_2 N_0 = 0.5 N_d V \quad (11)$$

and its standard deviation and coefficient of variation are:

$$\sigma_3 = \sqrt{\sigma_1^2 + P_2^2 \sigma_1^2 + \sigma_2^2} = \sqrt{1.25 \sigma_1^2 + \sigma_2^2} = \sqrt{1.25 N_d V + 0.125 N_d V} = \sqrt{1.375 N_d V} \quad (12)$$

$$CV_3 = \sqrt{5.5} \frac{1}{\sqrt{N_d V}} \quad (13)$$

The coverage interval (90% percentile interval) is:

$$N_i \in [N_3 - 1.645\sigma_3; N_3 + 1.645\sigma_3] \quad (14)$$

Eqs. (11)–(13) directly describe the statistical distribution of the number of fibres intersecting a given cross-section as a function of fibre dosage and fibre length. This formulation therefore links the stochastic mixing process at the volumetric level to the mechanical behaviour observed at the fracture section. It follows directly from Eq. (13) that the coefficient of variation decreases with increasing fibre dosage and increasing cross-sectional area, which explains the experimentally observed size effect in the scatter of bending tests.

The model demonstrates that the large scatter observed in standard beam tests is not primarily a material inconsistency, but a statistical consequence of limited fracture surface area and stochastic fibre dispersion.

3 Cross-section-dependent scatter due to fibre moment

In bending beam tests, not only the number of fibres intersecting the cracked section, but also their distance from the compressed zone of the cross-section is relevant (Bernard and Xu, 2017; Enfedaque et al, 2019). Even with the same number of fibres intersecting the cracked cross-section, the load–CMOD curves derived from FRC bending beam tests can exhibit a significant difference if the location of the fibres is different – for instance, if one sample has more intersecting fibres closer to the compressed zone than the other sample. In particular, the cross-section having more fibres closer to the compressed zone has a smaller moment capacity than that having more fibres farther from the cross-section, as shown in Fig. 3.

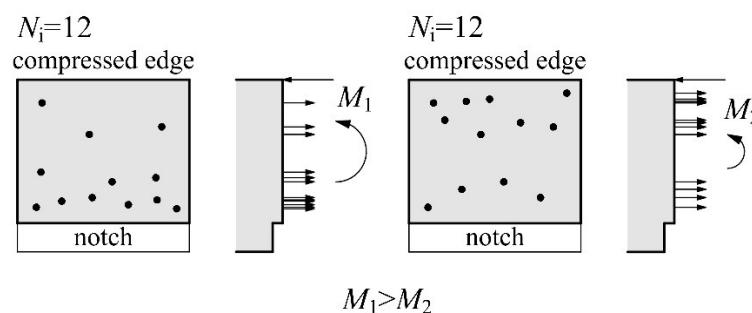


Figure 3: The effect of fibre position on the moment capacity of beams

After completing the beam test, the number and position of fibres on the fracture surface can be easily identified; however, determining the anchorage length and orientation of the fibres requires more advanced analysis. To parameterize the spatial distribution of fibres, Juhász introduced the concept of the fibre-moment (Juhász, 2015, 2020), which can be determined relatively easily and provides significant information about the tested beam. While the relationship between the number of fibres and the residual strength has been widely discussed in the literature (Barros et al, 2005; Lappa, 2007; Zerbino et al, 2012), only a few studies have considered the distance of fibres from the compressed zone (Bernard and Xu, 2017).

Based on this method, both the number of fibres in the cross-section and their distance from the compressed extreme fibre are analysed. Although the fibre orientation angle and anchorage length are not explicitly considered, previous research has demonstrated that the fibre-moment maintains a strong linear correlation with the residual strength (Juhász, 2018, 2020). The total fibre force distributed across the section is treated as a smeared tensile stress, consistent with established design recommendations, where this stress is assumed to remain approximately constant (RILEM, 2003; ACI, 1998; *fib* Model Code, 2012). As a result, it is reasonable to assume uniform fibre force within the cracked section, allowing the fibre-moment to be interpreted as a first-order moment. This parameter is unique to each tested beam and captures the spatial distribution of fibres across the section. The number of fibres is determined through visual inspection, which inherently introduces a degree of measurement uncertainty. However, in similar investigations, the discrepancy between fibre counts obtained via image analysis and manual counting was found to be negligible (Medeghini et al, 2025).

For precise determination, the fibre-moment can be expressed as follows:

$$S_{f, \text{test}} = \sum_{s=1}^{N_i} z_s N_{i,s} \quad (15)$$

where z_i is the distance associated with the fibre i from the compressed extreme fibre of the beam, with the thickness of the compressed concrete zone being neglected (see Fig. 4a).

The fibre-moment can also be calculated using a simplified approach by dividing the cross-section into horizontal bands (see Fig. 4b). Sensitivity analysis has shown that using five bands provides sufficient accuracy for practical purposes (Juhász, 2023). In this case, the fibre-moment can be determined as follows:

$$S_{f, \text{test}} = \sum_{j=1}^5 t_j N_{i,j} \quad (16)$$

where t_j and $N_{i,j}$ is according to Fig. 4b.

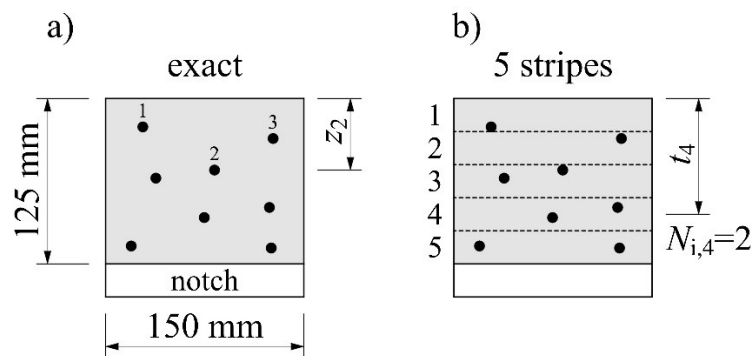


Figure 4: Determination of fibre-moment
(a) accurate calculation of fibre-moment (b) determination of fibre-moment using the band method.

The fibre-moment and residual strength values for individual beams are closely aligned along a straight line with a correlation coefficient $\delta > 0.9$, which allows for the elimination of variability due to the distribution of fibres during the evaluation of material parameters (Fig. 5). This strong correlation was observed for both steel and synthetic macrofibers, at both lower

and higher dosage levels, and across all residual strength parameters (Juhász, 2015, 2018; Tóth et al, 2017). A detailed description of the evaluation method can be found in (Juhász, 2020).

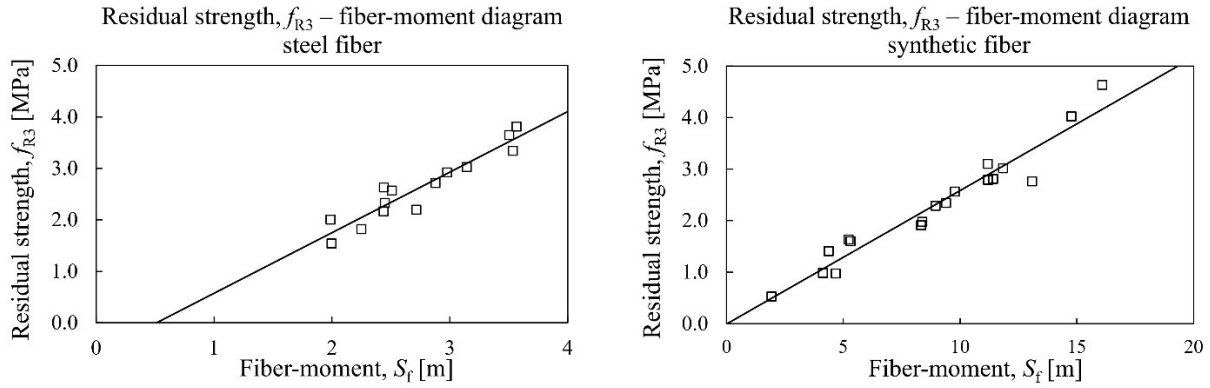


Figure 5: Residual strength ($f_{R,3}$) and fibre-moment (S_f) correlation diagram in case of (a) steel and (b) synthetic macrofibre

Assuming a perfectly homogeneous distribution, the centroid of the intersecting fibres coincides with the centroid of the cross-section. Under these conditions, an ideal fibre-moment can be defined:

$$S_{f,ideal} = N_i \frac{h}{2} \quad (17)$$

4 Improvement of beam test evaluation

The fibre-moments of the test specimens ($S_{f,j}$) and the corresponding residual strength ($f_{R,i,j}$) are approximately directly proportional, in other words, a larger fibre-moment corresponds to a larger residual strength. Consequently, a linear function can be fitted to the data of the complete set with linear regression, which is known as the reference function:

$$r(S_f) = a S_f + b \quad (18)$$

where:

$$a = \frac{\sum_{j=1}^n (S_{f,j} - S_{f,m,t})(f_{R,i,j} - f_{R,i,m})}{\sum_{j=1}^n (S_{f,j} - S_{f,m,t})^2} \quad (19)$$

$$b = f_{R,i,m} - S_{f,m,t} a \quad (20)$$

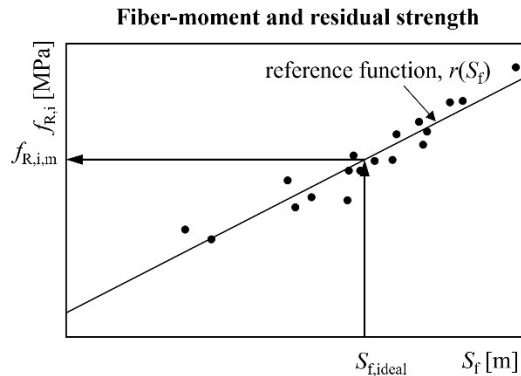


Figure 6: Mean residual strength evaluated from the reference function and ideal fibre-moment

If the effect of concrete is not considered, the values of the linear functions are expected to be approximately zero at the origin, which physically translates to the residual strength being zero at a fibre moment of zero.

If the expected number of fibres within the volume V is known, the mean value of the fibre moment can be inferred with the help of the ideal fibre-moment, Eq. (17). Subsequently, the mean residual strength ($f_{R,i,m}$) may be approximated by evaluating the reference function at this fibre moment (Fig. 6).

If the scatter of the results is interpreted as the distance from the reference curve, a characteristic reference curve may be defined by shifting the original reference curve. The value of the characteristic function evaluated at the mean fibre moment then represents the characteristic residual strength. A detailed description of this procedure is provided in (Juhász, 2020).

5 On-site verification of fibre content considering intrinsic scatter

The on-site verification of fibre quantity and mixing quality is of particular importance; therefore, based on the stochastic methods derived above, a testing procedure is presented that allows their assessment at a selected probability level. If the observed fibre count in a sample falls within this interval, the fibre dosage and mixing can be considered statistically acceptable.

The recommended procedure is as follows. From a given concrete mixer batch, three samples of volume V_t are taken, preferably from the beginning, middle, and end of the discharged concrete. In each sample, the number of fibres is counted, yielding $N_{t,1}$, $N_{t,2}$ and $N_{t,3}$. Two types of coverage intervals are then defined: one for the verification of fibre quantity and another for the assessment of mixing homogeneity.

For the verification of fibre quantity, the expected value and the standard deviation are calculated based on Eqs. (3) and (7):

$$N_t = N_d 3V_t \quad (21)$$

$$\sigma_t = \sqrt{N_d 3V_t} \quad (22)$$

Based on these values, the 90% probability interval is subsequently defined as:

$$(N_{t,1} + N_{t,2} + N_{t,3}) \in [N_t - 1.645 \sigma_t; N_t + 1.645 \sigma_t] \quad (23)$$

If this condition is satisfied, the fibre dosage and mixing can be considered compliant at the selected 90% reliability level.

To assess the quality of mixing, the counted values are evaluated separately in a similar manner:

$$N_{t,m} = N_d V_t \quad (24)$$

$$\sigma_{t,m} = \sqrt{N_d V_t} \quad (25)$$

$$N_{t,i} \in [N_{t,m} - 1.645 \sigma_{t,m}; N_{t,m} + 1.645 \sigma_{t,m}] \quad i = 1, 2, 3 \quad (26)$$

For the volume, the use of a standard 150 mm cube mould, or a formwork element of comparable dimensions, is recommended.

6 Concluding remarks

This paper investigates the scatter observed in fibre-reinforced concrete, which can be decomposed into intrinsic scatter and cross-section-dependent scatter. In both cases, the coefficient of variation decreases with increasing cross-sectional size and fibre dosage. This explains why the scatter of the standard small-scale three-point bending beam test (3PBBT) is particularly high and why it often results in unfavourable characteristic values. For fibre-reinforced concrete, the consideration of size effects therefore becomes an even more critical factor in the definition of material parameters.

The present study introduces the mechanisms responsible for the observed scatter and their theoretical derivation, and introduces novel evaluation approaches for the standard 3PBBT, as well as methods for defining acceptable intervals for on-site verification of fibre dosage. The paper further proposes a procedure, based on the derived statistical methods, for assessing on-site mixing quality and the quantity of dispersed fibres at a given reliability level.

It is evident that, from both an evaluation and a testing perspective, fibre-reinforced concrete must be conceptually decomposed into the concrete matrix and the fibres. In the case of fibre-reinforced concrete, the number of fibres is a key governing parameter, alongside the residual strength value.

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